Christ Church
Grammar School

2014

EPW 2
IN-CLASS VALIDATION

MATHEMATICS 3C/3D
Calculator-assumed

Your name _____

Time allowed for this section
Working time for this section: forty-five (45) minutes

Materials required/recommended for this section
To be provided by the supervisor
Nil

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: up to three calculators approved for use in the WACE examinations

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Calculator-assumed **(45 marks)**

Answer all questions.

Working time: 45 minutes.

Question 1 **(3 marks)**

By considering the expansion of $(a - b)^2$, prove that the sum of squares of two real numbers is always greater than or equal to twice their product.

Hint: Let the two real numbers be a and b.

Question 2 **(4 marks)**

The following pairs of fractions produce the same result if they are added together as when they are multiplied together.

$\frac{7}{2}$ and $\frac{7}{5}$ $\frac{11}{4}$ and $\frac{11}{7}$

$\frac{21}{11}$ and $\frac{21}{10}$ $\frac{13}{5}$ and $\frac{13}{8}$

$\frac{19}{7}$ and $\frac{19}{12}$ $\frac{72}{55}$ and $\frac{72}{17}$

These pairs of fractions are all in the form $\frac{k}{m}$ and $\frac{k}{n}$

(a) State the relationship that is shown between the numerator k and the denominators m and n. (1 mark)

(b) For any pair of fractions $\frac{k}{m}$ and $\frac{k}{n}$ where k has this relationship with m and n, prove that $\frac{k}{m} \times \frac{k}{n}$ will produce the same result as $\frac{k}{m} + \frac{k}{n}$ (3 marks)

Question 3 **(7 marks)**

(a) Write down the values of $m^2 + 7$ for $m = 1, 3, 5, 7$ and 9. (2 marks)

(b) By looking at your result for (a), state the largest integer, p, that $m^2 + 7$ is always divisible by, when m is a positive odd integer. (1 mark)

(c) Prove that $m^2 + 7$ is always divisible by p when m is a positive odd integer.

Hint: Let $m = 2k + 1$, where k is an integer. (4 marks)

Question 4 **(4 marks)**

Consider the following conjecture:
"Every prime number greater than 3 is one more or less than a multiple of 6."

(a) Show that the conjecture is true for three different prime numbers. (1 mark)

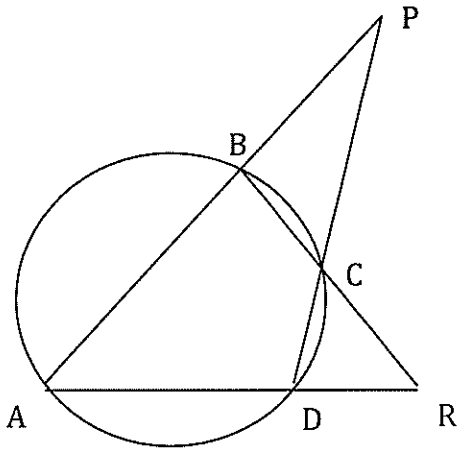
(b) By considering that any counting number can be written in the form $6n + p$, where n is an integer and p is an integer between 0 and 5, prove the above conjecture.

Hint: Consider each of $6n + p$, for $p = 0, 1, 2, 3, 4, 5$. (3 marks)

Question 5 **(4 marks)**

Consider the diagram below (not drawn to scale) which shows a cyclic quadrilateral ABCD. The sides of the quadrilateral have been extended and these lines meet at the points P and R as shown.

$$\angle ARB = 60° \text{ and } \angle BCP = 40°$$

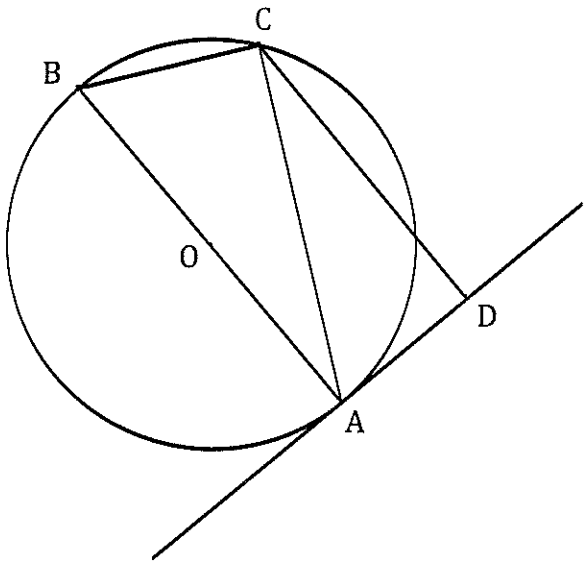


(a) Find $\angle ABC$. (2 marks)

(b) Hence find $\angle BAD$. (2 marks)

Question 6 **(6 marks)**

In the diagram below (not drawn to scale), AOB is the diameter of the circle, AC is a chord of the circle, AD is a tangent to the circle at A and CD is perpendicular to AD.



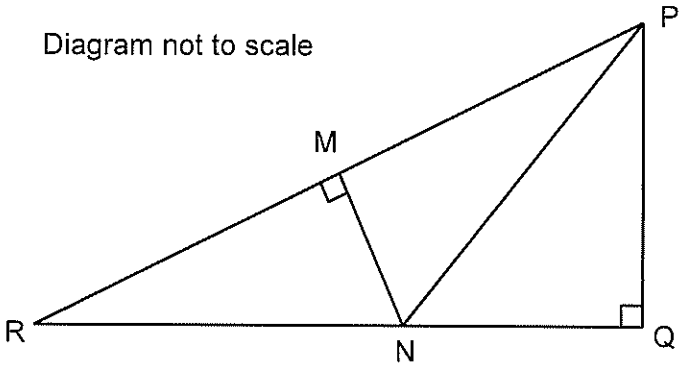
(a) Prove that ΔABC is similar to ΔCAD. (3 marks)

(b) Hence show that $AC^2 = AB \times CD$. (1 mark)

(c) Determine the radius of the circle when $AC = 15$ cm and $AD = 12$ cm. (2 marks)

Question 7 **(7 marks)**

In the diagram below, PQR is a right-angled triangle with $\angle PQR = 90°$ and M is the midpoint of PR. N is the point where the perpendicular to PR at M meets QR.



(a) Prove that ΔPNM is congruent with ΔRNM. (3 marks)

(b) If PN bisects $\angle QPR$, prove that ΔMPN is congruent with ΔQPN. (2 marks)

(c) By considering the results for (a) and (b), determine the ratio of the area of ΔPQN : area of ΔPQR . (2 marks)

End of questions